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MORSE-BOTT THEORETICAL SETTING FOR THE SEIBERG-WITTEN 4-DIM THEORY

CELSO MELCHIADES DORIA
UFSC - DEPTO. DE MATEMÁTICA

Let (X, g) be a closed, smooth riemannian 4-manifold. For any fixed spin^c structures α on X, the Seiberg-Witten functional

$$\mathcal{SW}_\alpha(A,\phi) = \int_X \{\frac{1}{2} \mid F_A^+ \mid^2 + \mid \nabla^A\phi \mid^2 + \frac{1}{4}k_g \mid \phi \mid^2 + \frac{1}{8} \mid \phi \mid^4\} dv_g \tag{0.1}$$

satisfies the Palais-Smale condition. There are two classes of critical points for the $\mathcal{SW}_\alpha$-functional (i) irreducibles: (A,ϕ), $\phi \neq 0$, (ii) reducibles: $(A, 0)$. For the purpose of studying smooth invariants on X, it only matters the existence of irreducible stable critical points of $\mathcal{SW}_\alpha$ ($\mathcal{SW}_\alpha$-monopoles) which exist only for a finite set of spin^c classes named *basic classes.* If the scalar curvature satisfies $k_g \geq 0$, then there is no irreducible critical points. The motivation to set up the $\mathcal{SW}_\alpha$-functional in a Morse-Bott theoretical framework is to understand the existence of $\mathcal{SW}$-monopoles from a analytical point of view, since in the presence of a $\mathcal{SW}_\alpha$-monopole the Morse-Bott index of the reducibles is greater than 0 . In order to achieve transversality conditions, the following perturbation of the Seiberg-Witten functional is considered: let η be a closed, smooth self-dual 2-form;

$$\begin{aligned}\mathcal{SW}^\eta_\alpha(A,\phi) = &\int_X \{\frac{1}{2} \mid F_A^+ \mid^2 + \mid \nabla^A\phi \mid^2 + \frac{1}{4}k_g \mid \phi \mid^2 + \frac{1}{8} \mid \phi \mid^4\} dv_g + \\ &- \int_X \left[< F_A^+ - \sigma(\phi), \eta > + \frac{1}{2} \mid \eta \mid^2 \right] dv_g.\end{aligned} \tag{0.2}$$

It is shown that for a large set of self-dual closed 2-forms η, the $\mathcal{SW}^\eta_\alpha$ functional fits into a Morse-Bott framework. The reducibles critical points define a critical set diffeomorphic to the jacobian torus $\mathcal{J}_X = H^1(X,\mathbb{R})/H^1(X,\mathbb{Z})$. The 2^{nd} variation formula (hessian) of $\mathcal{SW}^\eta_\alpha$ is obtained and the Morse-Bott index of reducible solutions $(A, 0)$ is shown to be the dimension of the largest negative eigenspace of the elliptic linear operator $L_{A,\eta} = \triangle_A + \frac{k_g}{4} + \eta$, hence is finite. Moreover, for a large set of self-dual closed 2-forms η, it is shown that the hessian's null space is exactly the tangent space to $\mathcal{J}_X$. In [1], they prove the gradient flow lines always converge to a critical point allowing to define a sort of Floer Complex. By using the blow-up ideas of Kronheimer-Mrowka in [2], it is possible to define Floer Homology Groups $\widehat{HF}(X;\alpha)$, $\check{HF}(X;\alpha)$ and $\overline{HF}(X;\alpha)$.

References

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E-mail address: cmdoria@mtm.ufsc.br